Renormalization group in dilute Bose gas: two faces of Janus

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Experimental and theoretical foundation

The great progress in experimental study of Bose – Einstein condensation of alkali metals ^{87}Rb, ^{23}Na, ^{7}Li in magnetic traps stimulated development of theory and revived attempts to prove the availability of a second order phase transition.

Already in 50s the non-relativistic homogeneous atomic Bose gas was studed at zero temperature in the frame of power expansion of $\sqrt{\rho a^3}$, where ρ is the gas density and a is S-wave scattering length. For typical alkali atoms a is defined from Van der Waals interaction and therefore it is much large than the Bohr radius; in the dilute gas we consider $\rho a^3 << 1$.

Problems arised at finite temperature, and long period the theory predicted a first order phase transition [1]-[2]. The existance of a second order phase transition was convinsingly prooved only in 90s with many-body t-matrix approximation [3]. Later it was demonstrated that this proof is most easy-obtained with renormalization group (RG) approach.

Nevertheless the using of RG contains inside a danger of misunderstanding because RG like Janus has more then one face.

Two faces of RG

The important and not very widely-known fact is existance of *two* mathematical approaches, named "renormalization group" alike [4].

One of them is "blocking" *Kadanov – Wilson semi-group* (KW), which initially emerged from the problem of spin lattice, and now is very popular in polymers, percolation etc. Its essence is overaging over some set of freedom degrees of the system.

Another is *Bogoliubov renormalization group*, which has more universal character and deals with an exact symmetry of the solution formulated in its "natural" relevant variables with suitable boundary conditions. It was created inside quantum field theory (and then its other name is *quantum field* or *field theoretical RG*); now it is applied to turbulence, continuous spin-field models, nonlinear optics and some others.

The glorious past: epoch of rival RGs

The RG approach applied to dilute Bose gas has a long history, and here we just point out the most brilliant results.

KW had very successful start: 3-dimensional Bose gas studied using knoledge of the microscopic details of the interaction between atoms, which influence on non-universal quantities. As a result the critical temperature T_c and the superfluid and condensate density of the Bose – Einstein condensed phase below T_c had been calculated [5].

Only later d-dimensional spin zero-Bose gas was investigated with field variant of RG at finite temperature [6]. The authors concentrated on the calculation of critical exponents. It was shown that the system undergoes a phase transition of the second order.

The presence of infrared divergences due to Goldstone mode is seems to be still the most serious problem of the theory.

The promising future: "field" face of RG in dilute Bose gas

There is a real necessity to face from KW "blocking" construction to the field theoretical Bogoliubov approach with S-action formulation. In the terms of the complex field ψ the action of a dilute Bose gas is constracted from the summarize the symmetries and interactions underlying the effective Langragian. It has the form [6]

$$S = \int_0^\beta d\tau \int d^d x \left[\frac{i}{2}\varepsilon_{ij}\psi_i\partial_\tau\psi_j + \frac{1}{4m}\nabla\psi_i\cdot\nabla\psi_i + \frac{g}{4}(\psi_i\psi_i)(\psi_j\psi_j)\right] \quad (\hbar\equiv 1) \ . \qquad (1)$$

Here $\beta\equiv 1/k_{\rm B}T$, the coupling constant $g = 2\pi a/m$.

Then we have the opportunity to constract the system of Schwinger – Dyson equations, perform the finite renormalization procedure and derive the RG equation. Let's note the main advantages of the Bogoliubov scheme:

— it has standard mathematical program how to treat a model — we need only to reformulate it for the case of dilute Bose gas;

— it allows (more easy then in KW) to solve the problem of Goldstone singularities with some redefinishion of fields.

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